THE WILCOXON'S MATCH PAIR TEST FOR FUZZY DATA

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The nonparametric statistical tests do not suposition about populational probability distributions. Between the several nonparametric statistical tests, the Wilcoxon Signed Rank Test is much utilized to compare small paired samples, with assumption level of measurement at least it is ordinal. In several experiments, the information can be of subjective in nature. For example, the experiment is compare the patient pain intensity before and after of he receive a painkiller. The patient can talk your concept of pain by one interval with a graduate scale by values from 1 to 10. Suppose that he answer a interval [3,5;5,5]; for the pain of feel patient. In several approachs, this information is take by statistic through of mean of interval. This approach is not incorrect, but it lost the important information: the interval weigh. This paper suggest a new methodology for this problem, using fuzzy numbers.

Formally, let X be a space of points, which a generic element of X denoted by x. A *fuzzy set* A in X is characterized by a *membership function* $\mu_A(x)$ which associates with each point in X a real number in the interval [0,1], [L. A. Zadeh, Fuzzy Sets. *Information and Control*, 8, 1965, 338-353]. A fuzzy number is a convex normalized fuzzy set M of the real line $\Re$, such that: (a) $\exists! x_0 \in \Re, \mu_M(x_0) = 1$ (x_0 is called the mean value of M); (b) μ_M is piecewise continuous. The four basic arithmetic operations on real numbers are extended to operations on fuzzy numbers, by extension principle and they can be found in [Dubois and Prade, H.; *Possibility Theory. An Approach to Computerized Processing of Uncertainty.* Plenum Press, New York, 1988]. The absolute value of a fuzzy number M, denoted by $abs(M)$, is: $abs(M) = M \cup (-M)$, if $M \in \Re^+$;

or 0, if $M \in \Re^-$; where $(-M)$ is opposite of the fuzzy number M and is such that $\mu_{(-M)}(x) = \mu_M(-x)$. We can note that M and $(-M)$ are symmetrical with respect to axis $x = 0$ and $abs(M)$ is positives. There are several forms of comparing and ordering fuzzy numbers. In this paper, we utilize an approach present by Dubois and Prade. Four index are proposed in: PS, NS, PSE and NSE, which are the possibility and necessity of strict exceedance and the possibility and necessity of exceedance or equallity, respectively.

Let be the same experiment as before and the good health conditions, before and after of treatment are fuzzy numbers, denoted by $\widetilde{x_i}$ and $\widetilde{y_i}$. Then, the set of paired data are done by $(\widetilde{x_i}, \widetilde{y_i})$. The Wilcoxon Signed Rank Test for Fuzzy Paired Samples proposed assume the fuzzy differences between N pairs of the observated data, that is, $\widetilde{d_i} = \widetilde{y_i} - \widetilde{x_i}$, for all i. As classical Wilcoxon Test, the fuzzy absolute value of differences must be order. In fuzzy samples, this is done according to $PS+NS$ index. We attach the sign of the d_i differences to the rank of each absolute value. Two rank sets are obtained: the ranks whose signs are negative $(R_k, 1 \leq k \leq K \leq N)$ and those with positive signs $(S_m, 1 \leq m \leq M \leq N)$. We can to note that S_m and R_k are not fuzzy values and statistic of the test is constructed by ranks, then we can use the same supositions as before and the distribution of ranks of this crisp ordering follows Wilcoxon signed-rank distribution under H_0 [Lehmann, E. L. and D'Abrera, H. J. M; *Nonparametrics: Statistical Methods Based on Ranks.* Holden-Day, San Francisco, 1975]. The statistic of the test is the same done by Lehmann and D'Abrera and the rejection of H_0 is indicated when V_S is sufficiently large, that is, $P(V_S \geq c) = \alpha$. When the two scores of any pair are equal or when two or more d_i's have the same size, the treatment is the same as the Classical Wilcoxon Test. When the data are done by crisp values, this new methodology return to classical test.

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